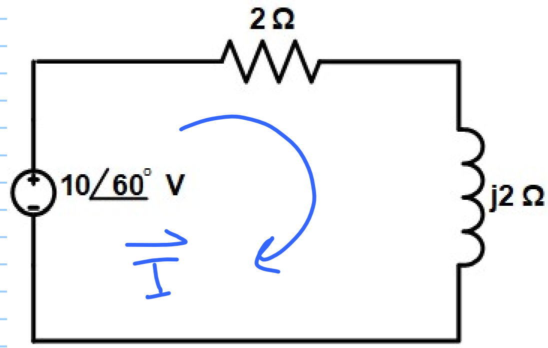
Example :

find the average power absorbed by each element.



$$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$P_{av.j2} = \text{Zero}.$$

$$\vec{I} = \frac{10\angle 60^\circ}{2+j2} = 3.53\angle 15^\circ \ A$$

$$\vec{V_R} = \frac{2}{2+j2} * 10\angle 60^\circ = 7.07\angle 15^\circ \ \text{Volt}.$$

OR $\vec{V_R} = \vec{I} R = 7.06\angle 15^\circ \ \text{Volt}.$

$$P_{av_R} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} * 7.07 * 3.53 \cos(15^\circ - 15^\circ)$$

$$= 12.47 \ W$$

$$P_{av_R} = \frac{I_m^2 R}{2} = \frac{1}{2} \times (3.53)^2 \times (2) = 12.46 \ W$$

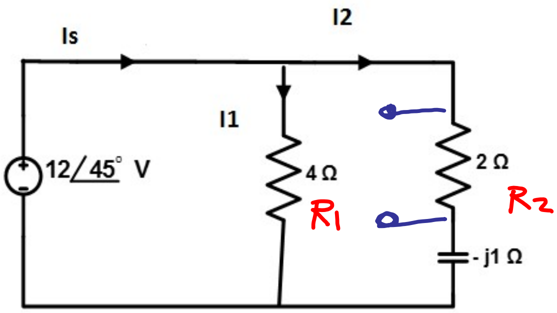
→ The average power supplied by the source

$$P_{av_{Vs}} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2}(10)(3.53)\cos(60^\circ - 15^\circ)$$

$$= 12.48 \ W$$

Example :



Determine the average power absorbed by each resistor . Determine the total average power supplied by the source .

$$\boxed{V_{R_2} = \frac{2}{2-j} * 12\angle 45}$$

$$P_{av_{R_1}} = \frac{V_m^2}{2R_1} = \frac{12^2}{2 \times 4} = 18\,W$$

$$\vec{I_2} = \frac{12\angle 45^\circ}{2-j} = 5.366\angle 71.56^\circ\ A$$

$$P_{av_{R_2}} = \frac{1}{2} I_{m_2}^2 R_2 = \frac{1}{2} \times 5.366^2 \times 2 = 28.79\,W$$

$$P_{av_{V_s}} = 18 + 28.79 = 46.79\,W$$

del.

OR

$$\vec{I_s} = \frac{\vec{V_s}}{Z_{eq}} = \vec{I_1} + \vec{I_2}$$

$$= \frac{12\angle 45}{4} + 5.366\angle 71.56$$

$$= 8.16\angle 62.1^\circ\ A$$

$$P_{av_{V_s}} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

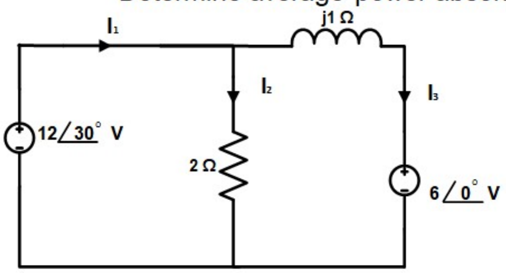
$$= \frac{1}{2} \times 12 \times 8.16 \times \cos(45 - 62.1)$$

$$= 46.785\,W$$

$$P_{abs.} = P_{del.}$$

Example :

Determine average power absorbed or supplied by each element .



$$\vec{I_2} = \frac{12}{2}\angle 30^\circ = 6\angle 30^\circ \text{ A}$$

$$\vec{I_3} = \frac{(12\angle 30^\circ) - (6\angle 0^\circ)}{\underset{1\angle 90^\circ}{j1}} = 7.43\angle -36.2 \text{ A}$$

$$\therefore \vec{I_1} = \vec{I_2} + \vec{I_3} = 6\angle 30^\circ + 7.43\angle -36.2 = 11.28\angle -7.08 \text{ A}$$

$$P_{av_{2\Omega}} = \frac{1}{2} I_{2m}^2 \times 2 = 36\text{W (absorbed)}$$

$$\frac{1}{2} I_m^2 R$$

$$P_{av_{j1\Omega}} = \text{Zero}$$

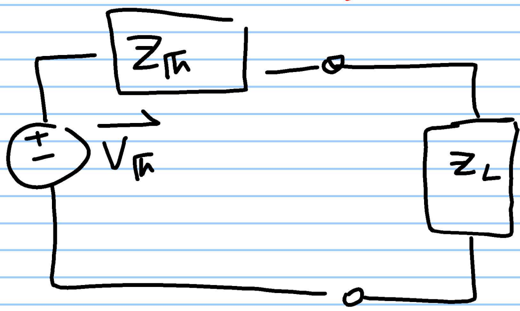
$$P_{av_{12\angle 30^\circ}} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} 12 \times 11.28 \cos(30 - (-7.08))$$

$$= 54\text{W (supplied)}$$

$$P_{av_{6\angle 0^\circ}} = \frac{1}{2} \times 6 \times 7.43 \cos(0 - (-36.2))$$

$$= 18\text{W (absorbed)}$$

Maximum Power Transfer



$$Z_{th} = R_{th} + j\,X_{th}$$

$$Z_L = R_L + j\,X_L$$

$$\longrightarrow \quad P_L = \frac{1}{2}\, I_{Lm}^2\, R_L$$

$$\vec{I} = \frac{\vec{V}_{th}}{Z_{th} + Z_L} = \frac{\vec{V}_{th}}{(R_{th}+R_L) + j(X_{th}+X_L)}$$

$$\therefore P_L = \frac{1}{2} \frac{V_{th}^2 \cdot R_L}{(R_{th}+R_L)^2 + (X_{th}+X_L)^2}$$

① $\frac{\partial P_L}{\partial X_L} = 0$ & ② $\frac{\partial P_L}{\partial R_L} = 0$

$$* \Rightarrow \frac{\partial P_L}{\partial X_L} = \frac{1}{2} \frac{-2 V_{th}^2 R_L (X_L + X_{th})}{\left[(R_{th}+R_L)^2 + (X_{th}+X_L)^2\right]^2}$$

for $\frac{\partial P_L}{\partial X_L} = 0 \longrightarrow \boxed{X_L = -X_{th}}$

$$* \Rightarrow \frac{\partial P_L}{\partial R_L} = \frac{V_{th}^2\left[(R_L+R_{th})^2 + (X_L+X_{th})^2 - 2R_L(R_L+R_{th})\right]}{2\left[(R_L+R_{th})^2 + (X_L+X_{th})^2\right]^2}$$

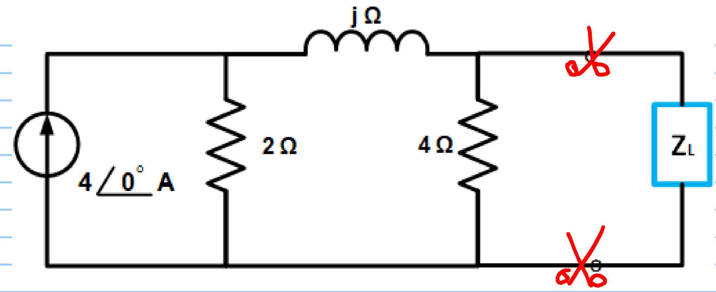
for $\frac{\partial P_L}{\partial R_L} = 0 \longrightarrow \boxed{R_L = R_{th}}$

$$\boxed{Z_L = Z_{th}^*} \qquad P_{L,max} = \frac{1}{8} \frac{V_{th}^2}{R_{th}}$$

$R_{th} = R_L$

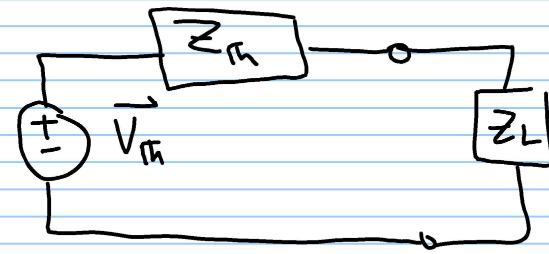
Example: Find Z_L for maximum average power transfer.

Compute the maximum average power supplied to the load.



$$Z_{th} = 4 // (2+j)$$

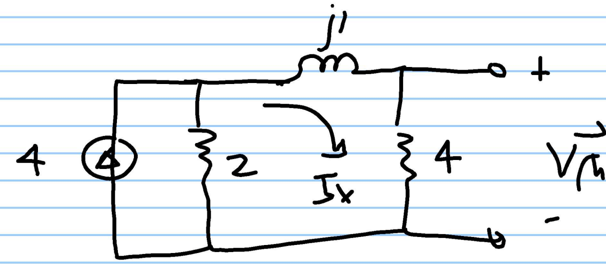
$$= 1.4 + j0.43\ \Omega$$



$$Z_L = Z_{th}^{*} = 1.4 - j0.43\ \Omega$$

$$P_{max} = \frac{1}{8}\,\frac{V_{th}^2}{R_{th}}$$

$$\vec{V}_{th} = \underbrace{\left(\frac{2\times 4}{6+j}\right)}_{I_x} \times 4$$



$$= 5.28\angle -9.46^\circ$$

$$\therefore\ P_{max} = \frac{1}{8}\,\frac{(5.28)^2}{1.4} = 2.489\ W$$

$$V_{TH} = 2I - 4\angle 0^\circ$$

$$I = \frac{V_X + 4\angle 0^\circ}{2+j4} \longrightarrow \quad I = \frac{-2I + 4}{2+j4}$$

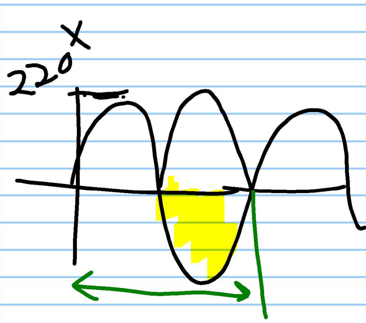
$$V_X = -2I$$

$$2I + j4I + 2I = 4$$

$$(4+j4)I = 4$$

$$I = 0.707\angle -\underline{\underline{45^\circ}}\ A$$

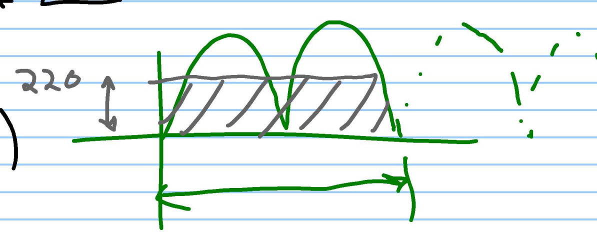
$$I = \frac{4}{\sqrt{4^2+4^2}\angle 45^\circ}$$



X

$$\underline{\underline{220}}\cos(2\pi\times 50t + 0^\circ)$$

$\underline{\underline{\text{rms}}}$



$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

eff

$$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

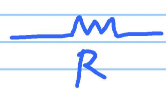
$$= V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

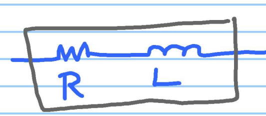
$$P_{av_R} = \frac{V_{rms}^2}{R} = I_{rms}^2 R = V_{rms} I_{rms}$$

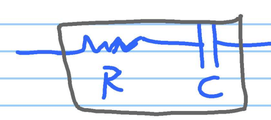
$$P_{av} = \underbrace{V_{rms} I_{rms}}_{P_a\ (VA)} \underbrace{\cos(\theta_v - \theta_i)}_{PF} \quad (W)$$

apparent Power — Power Factor

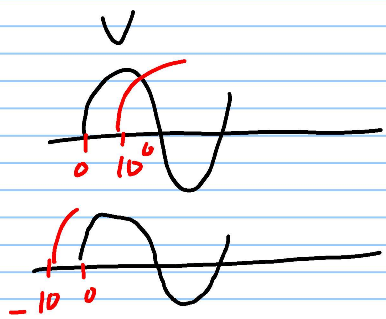
unity — lagging PF — leading PF

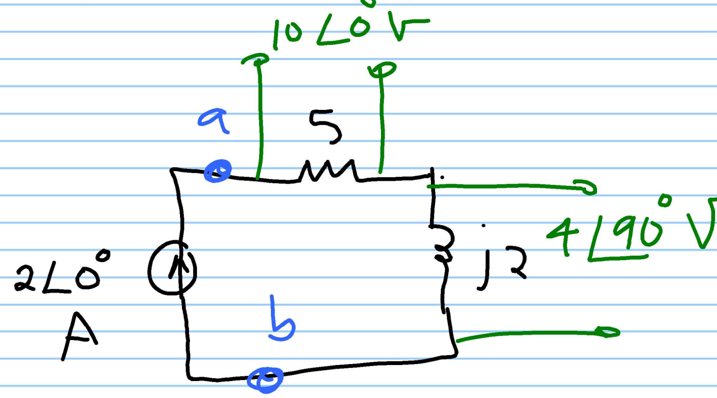






$\theta_v = \theta_i$ — i lags v by θ — i leads v by θ





$$\cos(\theta) = \cos(-\theta)$$

P.F. $= \cos(21.8) = 0.928$ lagging.

$$V_{ab} = (2\angle 0)(5 + j2)$$
$$= 10 + j4 = 10.77\angle 21.8^\circ$$